

Partial Differential Equations: Theory of Water Waves

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Course Description:

In these lectures we will start first with the theory of linear waves. Techniques of 19th century mathematics can be brought to bear with the linear waves. Then we will continue with the 20th century nonlinear water waves in which we will look at KdV, Burgers, NLS, flood and nerve waves problems. We will conclude by using the modern techniques of Inverse scattering, Hirotha method, singularity and Painlevé analysis. These techniques will be applied to interesting nonlinear models.

Prerequisites: ODEs, PDEs, Special Functions

Table of Contents

- Linear Waves [3]
 - Sound Waves
 - Plane Waves
 - Gravity Waves
 - Ship Waves
- Nonlinear Waves and Solitons
 - Solitary Waves [2], [1]
 - Elliptic Functions [2], [1]
 - Traffic Waves [3]
 - KdV, NLS [3], [1]
 - Flood Waves [1]
 - Nerves and Solitons [3], [1]
- Modern Perspectives - Theory of Water Waves
 - Variational Principle [1]
 - Hirotha Method [2]
 - Scattering Problem [2], [1], [5]
 - Painlevé analysis for ODEs [6]
 - Painlevé analysis for PDEs [7]
 - Singularity Analysis [4]

References:

- [1] Lokenath Debnath, *Nonlinear Partial Differential Equations*, (Birkhauser, Berlin, 1997)
- [2] P.G. Drazin and R.S. Johnson, *Solitons: an introduction*, (Cambridge University Press, Cambridge, 1996)
- [3] J. Billingham and A. C. King, *Wave motion*, (Cambridge University Press, Cambridge, 2000)
- [4] R. Conte, *Exact solutions of nonlinear partial differential equations by singularity analysis*, 2000, arXiv:nlin/0009024v1
- [5] Jianke Yang, *Nonlinear Waves in Integrable and Nonintegrable Systems* (SIAM, 2010)
- [6] R. Conte, *The Painlevé approach to nonlinear ordinary differential equations*, 1997, arXiv:solv-int/9710020v1
- [7] M. Musette, *Painlevé analysis for nonlinear partial differential equations*, 1998, arXiv:solv-int/9804003v1